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Demographic Aspects of Labour
Market Efficiency

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1. Intruduction

A general characteristic of a labour market is that the supply of labour and the demand for labour are unequal so that as a rule both, unemployment and vacant jobs can be observed at any point in time.

Economic theories of the labour market emphasise the importance of the wage rate and of the price of capital as the basic links between the supply of and the demand for labour. In the following analysis, however, wages and capital costs are not used as explanatory factors. The labour market will be regarded as a dynamic process and it will be attempted to identify the relevant elements of this process. In order to do this it has to be initially observed that the main function of the labour market is to attach individual workers to individual jobs. For the "attachement of workers to jobs" the term matching-process will be used.

The outcome of the matching-process on the supply side of the labour market is employment and unemployment and on the demand side occupied or vacant jobs. In order to explain these outcomes the matching-process itself has to be investigated. It is the matching-process which causes the various fluctuations in the status of a job (vacant, occupied) and in the status of those willing to work (employed, unemployed).

The variations in the status of (potential) employees are linked with the components of change of the labour stock due to the renewal process caused by retirement and natural increase. Similarly, the variations in the status of the jobs are linked with the components of change of the stock of jobs due to the renewal process caused by the depreciation of capital and by investment.

The aim of the analysis is to show that:

- (1) the changes in the status of (potential) employees and jobs are interdependent,
- (2) the intensity of the matching-process, measured by the number of changes in the status of jobs and (potential) employees, as well as

the efficiency of the matching-process are important factors for the explanation of the imbalances in labour markets.

- (3) the intensity as well as the efficiency of the matching-process are influenced by important demographic factors.

The main hypothesis of the analysis may be expressed as follows: Demographic factors such as the aging of the labour force, the decreasing size of the regional labour markets (caused by the death surplus and out-migration from big cities) and the increasing female labour participation rates are responsible for the diminishing efficiency of the matching-process and for the increasing imbalances in labour markets.

2. Description of the Outcomes of the Matching-Process

Considering a time period $[0, t]$, the magnitude of the labour force on a certain regional labour market at the beginning and at the end of the period is linked by the following equation

$$(1) \quad L(t) = L(0) - L_D(t) + L_N(t)$$

where $L(0)$ is the number of the (potential) employees at the beginning and $L(t)$ the number at the end of the period. $L_D(t)$ denotes those lost by retirement, by death, by out-migration and by commuting out of the region. $L_N(t)$ is the labour force gained by natural increase, by immigration and by commuting into the region. Equation (1) can be expressed in an alternative way by defining the difference $L(0) - L_D(t)$ as the stock present on the labour market at the beginning and at the end of period $[0, t]$, thus

$$(2) \quad L(t) = L_S(t) + L_N(t)$$

where $L_S(t) = L(0) - L_D(t)$.

The number of jobs A at the beginning and at the end of the period $[0, t]$ is linked by the analogous equation

$$(3) \quad A(t) = A(0) - A_D(t) + A_N(t)$$

where $A_D(t)$ is the number of jobs lost by the depreciation of capital and $A_N(t)$ is the number of jobs created by investment. For the number of jobs existing at the beginning and at the end of period $[0,t]$ the symbol $A_S(t)$ is used, so that equation (3) can be reformulated as

$$(4) \quad A(t) = A_S(t) + A_N(t)$$

where $A_S(t) = A(0) - A_D(t)$.

The three labour components $L_D(t)$, $L_S(t)$ and $L_N(t)$ and the three job components $A_D(t)$, $A_S(t)$, $A_N(t)$ are linked by variables describing the different possibilities of combination of the 6 components. For example: Among the number $A_D(t)$ of jobs lost in the period $[0,t]$ through the Depreciation of capital there is a subset of jobs which were occupied by a subset of the number $L_D(t)$ of employees who retired in the same period. The number of jobs in the subset of $A_D(t)$ and the number of employees in the subset of $L_D(t)$ which occupied these jobs are equal; this number is denoted by $Z_{DD}(t)$. The variable $Z_{DD}(t)$ is represented by the element in the first row and the first column of the matrix given in TABLE 1.

The system of notation for the variables in TABLE 1 is the following:

- F denotes the number of vacant jobs.
- U denotes the number of those unemployed.
- X is the symbol for the number of employees (which equals the number of occupied jobs).
- The first index of a variable refers to the row (labour), the second to the column (jobs).

T A B L E 1

Matrix of the State Variables of the Matching-
Process at the End of the Period [0,t]

	$A_D(t)$	$A_S(t)$	$A_N(t)$	
$L_D(t)$	$Z_{DD}(t)$	$F_{DS}(t)$	$F_{DN}(t)$	
$L_S(t)$	$U_{SD}(t)$	$x_{SS}(t)$	$x_{SN}(t)$	$U_S(t)$
$L_N(t)$	$U_{ND}(t)$	$x_{NS}(t)$	$x_{NN}(t)$	$U_N(t)$
		$F_S(t)$	$F_N(t)$	

Definitions:

number of jobs...

- $A_D(t)$... lost by capital depreciation in [0,t]
 $A_S(t)$... still existing at the end of period [0,t]
 $A_N(t)$... gained by investment in [0,t]

number of members of labour force...

- $L_D(t)$... lost by retirement, out-migration etc.
 $L_S(t)$... still existing at the end of period [0,t]
 $L_N(t)$... gained by natural increase, in-migration etc.

job-labour combinations

- $Z_{DD}(t)$ decrement of job-labour force combination A_D and L_D

$F_{DS}(t)$	<p>Number of members of the labour force and jobs respectively according to their status at the end of the period $[0,t]$</p> <p>Status of members of labour force: U = unemployed X = employed</p> <p>Status of jobs: F = vacant X = occupied</p> <p>The meaning of the Indices D, S and N is the same as for A and L above.</p>
$F_{DN}(t)$	
$U_{SD}(t)$	
$X_{SS}(t)$	
$X_{SN}(t)$	
$U_S(t)$	
$U_{ND}(t)$	
$X_{NS}(t)$	
$X_{NN}(t)$	
$U_N(t)$	
$F_S(t)$	
$F_N(t)$	

The variables in TABLE 1 represent the outcome of the matching process at a certain point of time, namely at the end of the period $[0,t]$. An individual member i of the labour force could be employed and unemployed several times during the whole period $[0,t]$. Thus, an individual belonging to the set $L_S(t)$ could pass through the following states:

<u>State</u>	<u>Time-Interval</u>	<u>Member of Labour Force i is a Member of Subset</u>	<u>Status</u>
1	d_1	$U_{SD}(d_1) = L_S(d_1) \cap A_D(d_1)$	unemployed
2	d_2	$X_{SS}(d_2) = L_S(d_2) \cap A_S(d_2)$	employed
3	d_3	$X_{SN}(d_3) = L_S(d_3) \cap A_N(d_3)$	employed
4	d_4	$U_S(d_4)$	unemployed
5	d_5	$X_{SS}(d_5) = L_S(d_5) \cap A_S(d_5)$	employed

In this example the individual i is a member of the group present on the labour market at the beginning and at the end of the period $[0,t]$: $i \in L(0)$ and $i \in L_S(t)$. The initial status of the

individual at $t = 0$ may be "employed", but the job history of the individual is assumed to change several times. In the example the individual loses occupation for the interval d_1 which is the first proportion of the period $[0, t]$. It is assumed that the individual loses occupation because his job disappears from the labour market due to the depreciation of capital. So the individual i belongs to the subgroup:

$$\text{State 1: } i \in U_{SD}(d_1) = L_S(d_1) \cap A_D(d_1)$$

which is the element in the second row and the first column of TABLE 1, assuming that TABLE 1 is set up for every time-interval d_1, d_2, \dots, d_5 , where $t = d_1 + d_2 + \dots + d_5$.

The second state in the job history is marked by a transition from the status "unemployed" (state 1) to the status "employed" which occupies the time interval d_2 . In the example it is assumed that the job occupied in time-interval d_2 belongs to the group $A_S(t)$ of jobs which are present on the labour market both at the beginning and at the end of period $[0, t]$:

$$\text{State 2: } i \in X_{SS}(d_2) = L_S(d_2) \cap A_S(d_2)$$

In state 3 the individual changes his job: the new job belongs to the group of jobs created in period $[0, t]$ denoted by $A_N(t)$:

$$\text{State 3: } i \in X_{SN}(d_3) = L_S(d_3) \cap A_N(d_3)$$

In the interval d_4 the individual is unemployed again and so is then to be included in the set U_S :

$$\text{State 4: } i \in U_S(d_4)$$

which contains all those who had occupied either a job among X_{SS} or a job among X_{SN} before becoming unemployed. An alternative way of arranging the notation is to split the variable U_S up into two variables U_{SS} and U_{SN}

$$U_S = U_{SS} + U_{SN}$$

where the second subscript of U_{SS} and U_{SN} denotes the group of jobs to which the lost jobs belong. In a similar way the variable U_N can be split into two variables U_{NS} and U_{NN} , where

$$U_N = U_{NS} + U_{NN}$$

The last state of the job history of individual i is state 5 in which the individual becomes reemployed by occupying a job among X_{SS} as he did in the time interval d_2 :

$$\text{State 5: } i \in X_{SS}(d_5) = L_S(d_5) \cap A_S(d_5)$$

According to the rule that each variable in TABLE 1 denotes a specific outcome of the matching-process at the end of period $[0, t]$, the individual i is finally attached to the group $X_{SS}(t)$ according to his status in the interval d_5 .

Similarly the fluctuations of the changes in the status of a certain job j belonging to $A_S(t)$ could be described, as an example, as follows:

<u>State</u>	<u>Time-Interval</u>	<u>Job j is an Element of Subset</u>	<u>Status</u>
1	g_1	$F_{DS}(g_1) = A_g(g_1) \cap L_D(g_1)$	vacant
2	g_2	$X_{NS}(g_2) = A_S(g_2) \cap L_N(g_2)$	occupied
3	g_3	$X_{SS}(g_3) = A_S(g_3) \cap L_S(g_3)$	occupied
4	g_4	$X_{NS}(g_4) = A_S(g_4) \cap L_N(g_4)$	occupied
5	g_5	$F_S(g_5)$	vacant

In this example the final status of job j is "vacant" and according to its last status the job is included in the variable $F_S(t)$ if g_5 is the last time interval in the history of the job. Similarly to the example above the variables F_S and F_N may be split into two sub-variables

$$F_S = F_{SS} + F_{NS}$$

$$F_N = F_{SN} + F_{NN}$$

Substituting the state variables of TABLE 1 into the balance equations (2) and (4) the following equations are obtained:

$$(5) \quad L(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t) + U_{SD}(t) + U_{ND}(t) + U_S(t) + U_N(t)$$

$$(6) \quad A(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t) + F_{DS}(t) + F_{DN}(t) + F_S(t) + F_N(t)$$

For a more comprehensive notation the following sums of components are defined:

$$(7) \quad X(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t)$$

$$(8) \quad U(t) = U_{SD}(t) + U_{ND}(t) + U_S(t) + U_N(t)$$

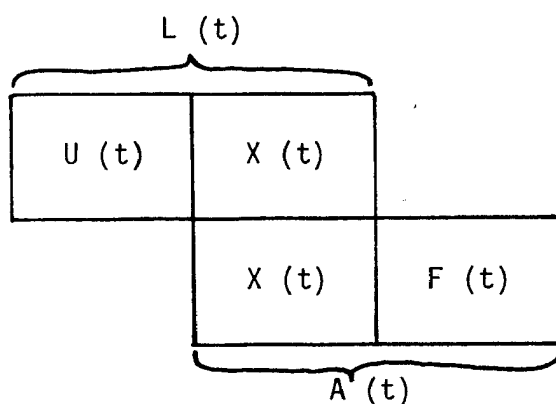
$$(9) \quad F(t) = F_{DS}(t) + F_{DN}(t) + F_S(t) + F_N(t)$$

With the definitions (7) - (9) the equations (5) and (6) can be rewritten as:

$$(10) \quad L(t) = X(t) + U(t)$$

$$(11) \quad A(t) = X(t) + F(t)$$

The normal situation of labour markets can be characterized as the coexistence of unemployed workers and vacant jobs, which is symbolically represented in the following diagram:



where $L(t)$ = total size of working force
 $U(t)$ = number of unemployed
 $X(t)$ = number of employed = number of occupied jobs
 $A(t)$ = total number of jobs
 $F(t)$ = number of vacant jobs.

Different kinds of imbalances on the labour market can now be distinguished:

(1) If the size of the labour force exceeds the number of jobs (i.e. $L(t) > A(t)$) the job deficit or labour surplus is

$$(11.1) \quad L(t) - A(t) = U(t) - F(t)$$

(2) If the number of jobs exceeds the size of the working force (i.e. $A(t) > L(t)$) the labour deficit or job surplus is

$$(11.2) \quad A(t) - L(t) = F(t) - U(t)$$

(3) If the number of jobs is equal to the size of the labour force (i.e. $L(t) = A(t)$) there is no job deficit and no labour deficit, but the coexistence of unemployed and vacant jobs still justifies a characterization of this case as a "disequilibrium on the labour market".

Clearly, the labour-surplus as well as the labour deficit are, to a considerable extent, due to demographic factors. But the aim of this paper is to show also that the third case, the zero-deficit-disequilibrium-case, is caused by demographic factors because these factors lead to a decline in the intensity and in the efficiency of the matching-process in many industrialized countries.

3. Theoretical Analysis of the Dynamic Characteristics of the Matching-Process. Definition of the Intensity and of the Efficiency of the Matching-Process

In the previous section examples for the changes in the status of an individual member of the labour force i and of the status of an indi-

vidual job j were discussed. The individual person i experienced the status "employed" three times and the status "unemployed" twice; the job j had the status "occupied" three times and the status "vacant" twice in the considered interval of time. Every individual and every job is attached to one of the variables $X(t)$, $U(t)$ or $F(t)$ only once, namely according its status at the end of the period $[0, t]$. Obviously the number of changes of the status of the members of the working force (jobs) during the whole period $[0, t]$ normally considerably exceeds the number of members of the working force (jobs).

In order to determine the difference between the size of the working force (jobs) and the number of their status changes two large classes of different kinds of job-labour-matching, "permanent-stoch-matching" (abbreviated by the symbol "PSM") and "renewal-matching" (abbreviated by "RM") are first distinguished.

Permanent-stock-matching takes place between jobs and workers who are present on the labour market at the beginning and at the end of the period $[0, t]$, i.e. between the individual jobs included in $A_S(t)$ and the individual members of the labour force included in $L_S(t)$ (TABLE 2).

Renewal-matching takes place between individual jobs included in $A_D(t)$ or in $A_N(t)$ and individual members of the labour force included in $L_D(t)$ or in $L_N(t)$ (TABLE 2).

T A B L E 2

Classification of Types of Matching-Processes

		$A_D(t)$	$A_S(t)$		$A_N(t)$
			$F_p(o)$	$x_p(o)$	
$L_D(t)$		RM_1	RM_2		RM_3
$L_S(t)$	$y_p(o)$	RM_4	Permanent Stock - Matching PSM		RM_5
	$x_p(o)$				
$L_N(t)$		RM_6	RM_7		RM_8

RM = Renewal-Matching

Firstly an analysis of permanent-stock-matching will be made. In order to do this, PSM is classified into 4 sub-classes. At the beginning of period $[0,t]$ individuals included in $L_S(t)$ are either employed (denoted by $X_p(0)$) or unemployed (denoted by $U_p(0)$) and the jobs included in $A_S(t)$ are either occupied (denoted by $X_p(0)$) or vacant (denoted by $F_p(0)$):

$$L_S(t) = X_p(0) + U_p(0)$$

$$A_S(t) = X_p(0) + F_p(0)$$

Matching can take place between

individuals in	$U_p(0)$	PSM case 1
and jobs in	$F_p(0)$	
individuals in	$U_p(0)$	PSM case 2
and jobs in	$X_p(0)$	
individuals in	$X_p(0)$	PSM case 3
and jobs in	$F_p(0)$	
individuals in	$X_p(0)$	PSM case 4
and jobs in	$X_p(0)$	

In TABLE 2 the 4 cases may be symbolized by splitting the PSM-field into 4 sub-fields.

For each case $i = 1, \dots, 4$ the following basic variables are defined: $y_i(t)$ denotes the number of state transitions from the state "employed" (denoted by the symbol "e") to the state "unemployed" (denoted by the symbol "u"), which may be abbreviated as $e \rightarrow u$. The variable $y_i(t)$ gives the number of state transitions per unit of time:

$$(12) \quad y_i = y_i(t)$$

The variable $v_i(t)$ defines the number of state transitions between the state "vacant" (abbreviated by the symbol "v") and "occupied" (symbol "o") for the jobs:

$$(13) \quad v_i = v_i(t)$$

The variable $m_i(t)$ is equal to the number of successful job-labour-matchings per unit of time:

$$(14) \quad m_i = m_i(t)$$

In case 1 the matching-process for an individual may lead to the following sequence of state transitions:

status of individual i	e, u, ...
status of the job concerned	o, v, ...

If three successive time intervals $\Delta t_1, \Delta t_2, \Delta t_3$ and three individual workers are considered then the job history of the three workers could be described by the following three sequences

	Δt_1	Δt_2	Δt_3
individual 1	e	u	-
job concerned	o	v	-
individual 2	-	e	u
job concerned	-	o	v
individual 3	e	u	e
job concerned	o	v	o

The characteristics of all sequences of case 1 are the following:
all sequences start with an e/o combination and end either with an e/o combination or an u/v combination. In the above example the variables m_1, y_1 and v_1 have the following values:

$$\left. \begin{array}{l} m_1(\Delta t_1) = 2 \\ m_1(\Delta t_2) = 1 \\ m_1(\Delta t_3) = 1 \end{array} \right\} \quad \text{measured by the number of all e/o combinations.}$$

$$\begin{aligned}
 y_1(\Delta t_1) &= 0 \\
 y_1(\Delta t_2) &= 2 \\
 y_1(\Delta t_3) &= 1 \\
 v_1(\Delta t_1) &= 0 \\
 v_1(\Delta t_2) &= 2 \\
 v_1(\Delta t_3) &= 1
 \end{aligned}$$

measured by the number of all
u/v combinations

In the three time intervals it happens that

$$\begin{aligned}
 m_1(\Delta t_1) &> y_1(\Delta t_1) = v_1(\Delta t_1) \\
 m_1(\Delta t_2) &< y_1(\Delta t_2) = v_1(\Delta t_2) \\
 m_1(\Delta t_3) &= y_1(\Delta t_3) = v_1(\Delta t_3),
 \end{aligned}$$

but as any sequence starts with an e/o combination and ends either with an e/o combination or an u/v combination, the relation for the sum of the three time intervals for all individuals is

$$\sum_i m_1(\Delta t_i) \geq \sum_i y_1(\Delta t_i) = \sum_i v_1(\Delta t_i)$$

As a general rule it can be stated for case 1 that the number of successful matchings for an extended time period $[0, t]$ is greater than the number of unemployment cases (which equals the number of vacancy cases):

$$(15) \quad \int_0^t m_1(t) dt \geq \int_0^t y_1(t) dt = \int_0^t v_1(t) dt$$

Contrary to case 1 all matching processes of the cases 2, 3 and 4 have individual sequences starting with an u/v combination, not by an e/o combination as in case 1, and they end either with an u/v combination or an e/o combination:

status of the individual i	u, e, \dots
status of the job concerned	v, o, \dots

Therefore in cases 2, 3 and 4 the number of successful matchings for an extended period $[0, t]$ is less than or equal to the number of unemployment cases (equal to the number of vacancy cases):

$$(16) \quad \int_0^t m_2(t)dt \leq \int_0^t y_2(t)dt = \int_0^t v_2(t)dt$$

$$(17) \quad \int_0^t m_3(t)dt \leq \int_0^t y_3(t)dt = \int_0^t v_3(t)dt$$

$$(18) \quad \int_0^t m_4(t)dt \leq \int_0^t y_4(t)dt = \int_0^t v_4(t)dt$$

To quantify the variables over the whole period $[0, t]$ the rates can be summed over the time intervals within $[0, t]$ which leads to the following corresponding definitions:

$$\left. \begin{aligned} (19) \quad Y_i(t) &= \int_0^t y_i(t)dt \\ (20) \quad V_i(t) &= \int_0^t v_i(t)dt \\ (21) \quad M_i(t) &= \int_0^t m_i(t)dt \end{aligned} \right\} \begin{array}{l} \text{for all} \\ \text{PSM cases} \\ i = 1, \dots, 4 \end{array}$$

For case 1 according to (15):

$$(22) \quad M_1(t) \geq Y_1(t) = V_1(t)$$

For cases 2, 3 and 4:

$$(23) \quad M_2(t) \leq Y_2(t) = V_2(t)$$

$$(24) \quad M_3(t) \leq Y_3(t) = V_3(t)$$

$$(25) \quad M_4(t) \leq Y_4(t) = V_4(t)$$

T A B L E 3

Effects of the Components of Change of the Stock of Members of the Labour Force and of the Stock of Jobs on the Variables $Y(t)$ and $V(t)$

	$A_D(t)$	$A_S(t)$	$A_N(t)$	
$L_D(t)$	no effect	increase of $V(t)$	increase of $V(t)$	
$L_S(t)$	increase of $Y(t)$	P S M - cases 1, 2, 3, 4	increase of $V(t)$	increase of $Y(t)$
$L_N(t)$	increase of $Y(t)$	increase of $Y(t)$	increase of $Y(t)$ and $V(t)$	increase of $Y(t)$
		increase of $V(t)$	increase of $V(t)$	

In order to take account of the effects of the different types of RM-matchings given in TABLE 2 on the number of job vacancies and on the number of unemployment cases the variables $V_{RM}(t)$ and $Y_{RM}(t)$ are defined so that the sum of job vacancies and unemployment cases of both the PSM- and the RM-matchings is

$$(30) \quad Y^*(t) = Y(t) + Y_{RM}(t) \quad , \quad Y_{RM}(t) \geq 0$$

$$(31) \quad V^*(t) = V(t) + V_{RM}(t) \quad , \quad V_{RM}(t) \geq 0$$

Correspondingly the total number of successful matchings is defined as the sum of the successful matchings of the PSM-class and of the RM-class:

$$(32) \quad M^*(t) = M(t) + M_{RM}(t) \quad , \quad M_{RM}(t) \geq 0$$

For equations (30), (31) and (32) it is not necessary to introduce the assumption of additivity because the additivity of the variables is guaranteed by their definition.

For the PSM-class it has been demonstrated that $Y(t)$ equals $V(t)$. But for the RM class the relation between $Y_{RM}(t)$ and $V_{RM}(t)$ is not known. As a consequence the relation between $Y^*(t)$ and $V^*(t)$ may be $Y^*(t) \geq V^*(t)$ or $Y^*(t) \leq V^*(t)$. Therefore an preliminary formulation for the total matching-function f is

$$(33) \quad M^*(t) = f(Y^*(t), V^*(t))$$

In order to specify the type of this function, the 4 PSM cases have to be examined. For three of the 4 cases it has been demonstrated that $M(t)$ is less than or equal to $V(t)$ ($= Y(t)$). Only for the class 1 does $M(t)$ exceed $V(t)$ and so $Y(t)$. Class 1 was characterized as the sum of matching-processes of those who are unemployed and of jobs which are vacant at the beginning of the processes. For the different types of the RM-classes given in TABLE 2 each of the variables $A_D(t)$, $A_N(t)$ can be split into two variables $F(t)$, $X(t)$ and each of the variables $L_D(t)$ and $L_N(t)$ into two variables $U(t)$ and $X(t)$. Thus for each of the

RM-cases RM_1, \dots, RM_g in TABLE 2 four sub-cases can be distinguished by combining the corresponding $F(t)$, $U(t)$ and $X(t)$ variables as has been done for the PSM-class. It can be concluded that the variable $M(t)$ exceeds $V(t)$ or $Y(t)$ only in approximately one fourth of all cases. As the coefficients $F(t)/X(t)$ and $U(t)/X(t)$, which determine the distribution of jobs and individuals in the 4 cases as well as the distribution of the matchings on the 4 cases, are small, it seems to be justified to assume that, in total, $M(t)$ does not exceed either $Y(t)$ or $V(t)$. An adequate specification of the total matching-function can therefore be given as

$$(34) \quad M^*(t) = \alpha Y^*(t) \quad \text{for } Y^*(t) < V^*(t)$$

$$(35) \quad M^*(t) = \alpha V^*(t) \quad \text{for } V^*(t) < Y^*(t)$$

or as

$$(36) \quad M^*(t) = \alpha \min(Y^*(t), V^*(t))$$

$$0 \leq \alpha \leq 1$$

The derivatives of the variables in (34) - (36) give the values of the variables per unit of time rather than for the whole period $0, t$:

$$(37) \quad y^*(t) = \frac{d}{dt} Y^*(t)$$

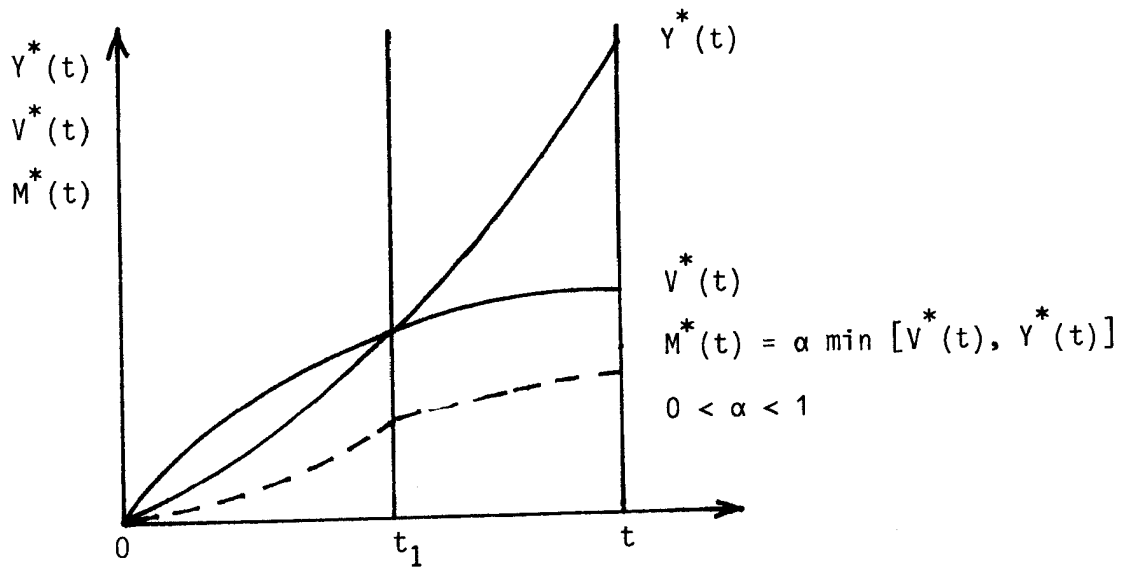
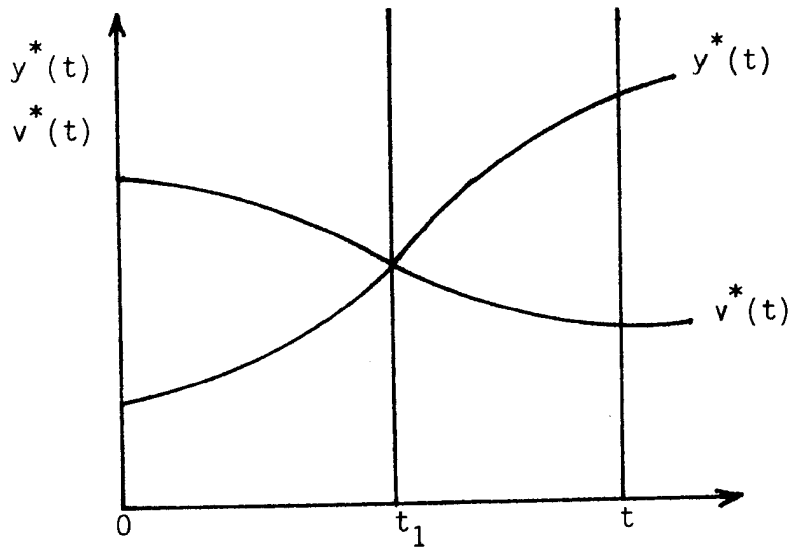
$$(38) \quad v^*(t) = \frac{d}{dt} V^*(t)$$

$$(39) \quad m^*(t) = \alpha \min(y^*(t), v^*(t))$$

An example of the time paths of these variables and of the matching-function is given in GRAPH 1 below.

G R A P H 1

Characteristics of the Matching-Function $M^*(t)$



4. The Diminishing Congruency of the Renewal Processes of the Labour Force and of the Capital Stock and its Effects on the Efficiency of the Matching-Process and on the Disequilibrium of the Labour Market

The question of how demographic change influences the imbalance in the labour market and the efficiency of the matching-process will now be considered. Empirical data for measuring the variables $Y(t)$, $V(t)$ and $M(t)$ are normally not available. But even without empirical evidence it seems to be possible to discuss important aspects of the problem on the basis of a minimum of plausible assumptions.

The first assumption is that $V^*(t)$ is less than or equal to $Y^*(t)$ so that the matching-function can be written as

$$(40) \quad M^*(t) = \alpha V^*(t) \quad , \quad 0 \leq \alpha \leq 1$$

The second assumption made is that the number of job vacancies $V^*(t)$ is, among other factors, a function of the number of jobs at the beginning of the period $[0, t]$, i.e.

$$(41) \quad V^*(t) = \beta A(0) \quad , \quad 0 \leq \beta \leq 1$$

The last and central assumption is that the change in the number of the employed ΔX is a function of the number of successful matchings:

$$(42) \quad \Delta X = \gamma M^*(t) \quad , \quad \gamma \leq 1$$

Since ΔX can be negative and $M^*(t)$ is greater than or equal to zero, the range for the parameter γ is $\gamma \leq 1$, whereas the range for α and β is $0 \leq \alpha, \beta \leq 1$. α determines the number of successful matchings and therefore it is called the "efficiency parameter". For the parameter β the term "intensity parameter" is suggested because β measures the relation between the number of fluctuations and the stock of jobs which is, among other factors, the main source of these fluctuations. For γ the term "employment parameter" is suggested.

If in the definitions

$$(43) \quad \Delta X = X(t) - X(0) = L(t) - U(t) - (L(0) - U(0))$$

$$= \Delta L - \Delta U$$

and

$$(44) \quad \Delta X = X(t) - X(0) = A(t) - F(t) - (A(0) - F(0))$$

$$= \Delta A - \Delta F$$

the ΔX from equation (42) is substituted, the increase of the number of unemployed and of the number of vacant jobs is obtained as a function of the parameter γ :

$$(45) \quad \Delta U = \Delta L - \gamma M^*(t)$$

$$(46) \quad \Delta F = \Delta A - \gamma M^*(t)$$

Using equation (40) and (41) in (45) and (46)

$$(47) \quad \Delta U = \Delta L - \alpha\beta\gamma A(0)$$

$$(48) \quad \Delta F = \Delta A - \alpha\beta\gamma A(0)$$

are obtained.

As a consequence of the three assumptions it can be stated that the higher the intensity and the efficiency of the matching-process the lower - ceteris paribus - is the increase in unemployment and job vacancy. This result is quite plausible, but it has been deduced on the basis of the condition that the matching-parameters are constant.

The condition of constancy will now be replaced by other assumptions which can be derived on the basis of the results of Section 3.

For a determination of the efficiency parameter α the following function is suggested:

$$(49) \quad \alpha(t) = \alpha (L_D(t) - A_D(t), L_S(t) - A_S(t), L_N(t) - A_N(t))$$

The arguments in the function of equation (49) are the differences between pairs of various components of change of the number of members of the labour force and of the number of jobs in the period $[0, t]$. The idea of the specification of (49) is the following: The renewal of the labour force and of the capital stock are interpreted to be two interdependent processes. Interdependency means that the components $L_D(t)$ and $A_D(t)$ and the components $L_N(t)$ and $A_N(t)$ are linked by the matching-process so that the variables in each pair of components should be of a similar magnitude.

In fact the figures for both processes show a high similarity between the age of the capital stock and the age of the labour force: A young labour force is, as a general rule, associated with a high proportion of new capital equipment. Figures for the censuses of 1961 and 1970 in the Federal Republic of Germany show a strong similarity of the components of change in both processes¹⁾:

Components of Change in the Labour Force		Components of Change in the Stock of Jobs	
L	(61) = 26,8 Mill.	A	(61) = 26,5 Mill.
L_D	(61-70) = 12,6 Mill.	A_D	(61-70) = 12,0 Mill.
L_S	(61-70) = 14,2 Mill.	A_S	(61-70) = 14,5 Mill.
L_N	(61-70) = 12,3 Mill.	A_N	(61-70) = 11,8 Mill.
L	(70) = 26,5 Mill.	A	(70) = 26,3 Mill.

L_D (61-70) is 12.600.000, including deaths, retirement and out-migration. Although L_D (61-70) is made up of very different types of components, its magnitude is approximately that of A_D (61-70).

1) Sources for these figures can be found in: H. Birg, "An Inter-regional Population Employment Model for the Federal Republic of Germany: Methodology and Forecasting Results for the Year 2000" in: Papers of the Regional Science Association, Vol. 47, 1981, P. 97f.

Also of similar magnitude are the components of L_N (61-70) and A_N (61-70), although the component L_N is a heterogeneous composition of three different groups of persons - young people who enter the labour market by aging, foreign workers who immigrated into the Federal Republic during 1961-70 (about 6 million) and women who entered the labour market following a change in the female activity rate. The component L_N is of similar size to the number of new jobs A_N . As a consequence of the similarity of L_D and A_D and of L_N and A_N , the permanent stock of the labour force and the permanent stock of jobs is also of approximately the same magnitude.

What is the reason for this similarity? It would seem to be plausible that the occupational skills of an older employee do not fit the needs of a new job to the same extent as the skills of a younger employee and it seems also plausible that young people have a preference for new jobs in young industries. Therefore the probability of successful matching is higher the better the characteristics of the members of the labour force, expressed by their average age, fit the characteristics of jobs, expressed by the age of the capital stock. The pairwise differences between the components of change of the stock of the labour force and of the components of change of the stock of jobs can be interpreted as proxy variables for the similarities in the characteristics of employees and jobs.

If the assumption is accepted that the higher the similarity between the components of the labour capital renewal processes the higher is the efficiency of the matching-process, then the differences $L_D - A_D$, $L_S - A_S$ and $L_N - A_N$ are appropriate variables in the function of equation (49) which determines α . These assumptions may be expressed in terms of the partial derivatives of α as:

$$(50.1) \quad \frac{\partial \alpha}{\partial (L_D - A_D)} < 0$$

$$(50.2) \quad \frac{\partial \alpha}{\partial (L_S - A_S)} < 0$$

$$(50.3) \quad \frac{\partial \alpha}{\partial (L_N - A_N)} < 0$$

implying that the lower the differences in matching the higher its efficiency is.

For the intensity parameter β the following function β is suggested:

$$(51) \quad \beta(t) = \beta \left(\frac{L_D(t)}{L(0)}, \frac{L_N(t)}{L(0)}, \frac{A_D(t)}{A(0)}, \frac{A_N(t)}{A(0)} \right)$$

The assumption made in equation (51) is that the magnitudes of the renewal components of individuals and jobs in relation to the magnitudes of their initial stocks determine, among other factors, the number of job vacancies $V^*(t)$, given the number of jobs and members of the labour force at the beginning of the process. The assumption can be expressed in the following way:

$$\left. \begin{aligned} (52.1) \quad \frac{\partial \beta}{\partial (L_D(t)/L(0))} &> 0 \\ (52.2) \quad \frac{\partial \beta}{\partial (L_N(t)/L(0))} &> 0 \end{aligned} \right\} \quad \text{- demographic influence}$$

$$\left. \begin{aligned} (52.3) \quad \frac{\partial \beta}{\partial (A_D(t)/A(0))} &> 0 \\ (52.4) \quad \frac{\partial \beta}{\partial (A_N(t)/A(0))} &> 0 \end{aligned} \right\} \quad \text{- economic influence}$$

In (52) the first pair of factors is due to a genuine demographic influence, the second pair is associated with economic influence, but both are assumed to be highly interdependent as demonstrated in the explanation for equation (49).

How can the condition of constancy for the parameter γ , introduced in (42), be replaced by a more realistic assumption? This is a central question because answering it implies establishing a hypothesis of the determinants of the change in employment. The following hypothesis is suggested:

$$(53) \quad \gamma(t) = \gamma(\alpha(t), \beta(t), D(t), p_L(t)/p_C(t))$$

whereby

$$(53.1) \quad \frac{\partial \gamma}{\partial \alpha} > 0, \quad \frac{\partial \gamma}{\partial \beta} > 0$$

$$(53.2) \quad \frac{\partial \gamma}{\partial D} > 0, \quad \frac{\partial \gamma}{\partial (P_L/P_C)} < 0$$

In (53) $D(t)$ is the expected demand for the products of the firms. $P_L(t)$ and $P_C(t)$ are the prices per unit of labour and per unit of capital respectively. The demand variable $D(t)$ and the relation between the prices per unit of labour and capital $P_L(t)/P_C(t)$ are explanatory factors commonly used in employment theory.

By introducing the matching process parameters α and β in hypothesis (53) a step towards a more endogenous explanation of the labour-market balance can be made: As shown, α and β are originated - to a considerable extent - by demographic change associated with the renewal process within the labour force. Introducing α and β in function (53) means that employment change is explained not only by economic variables but also by genuine demographic factors.

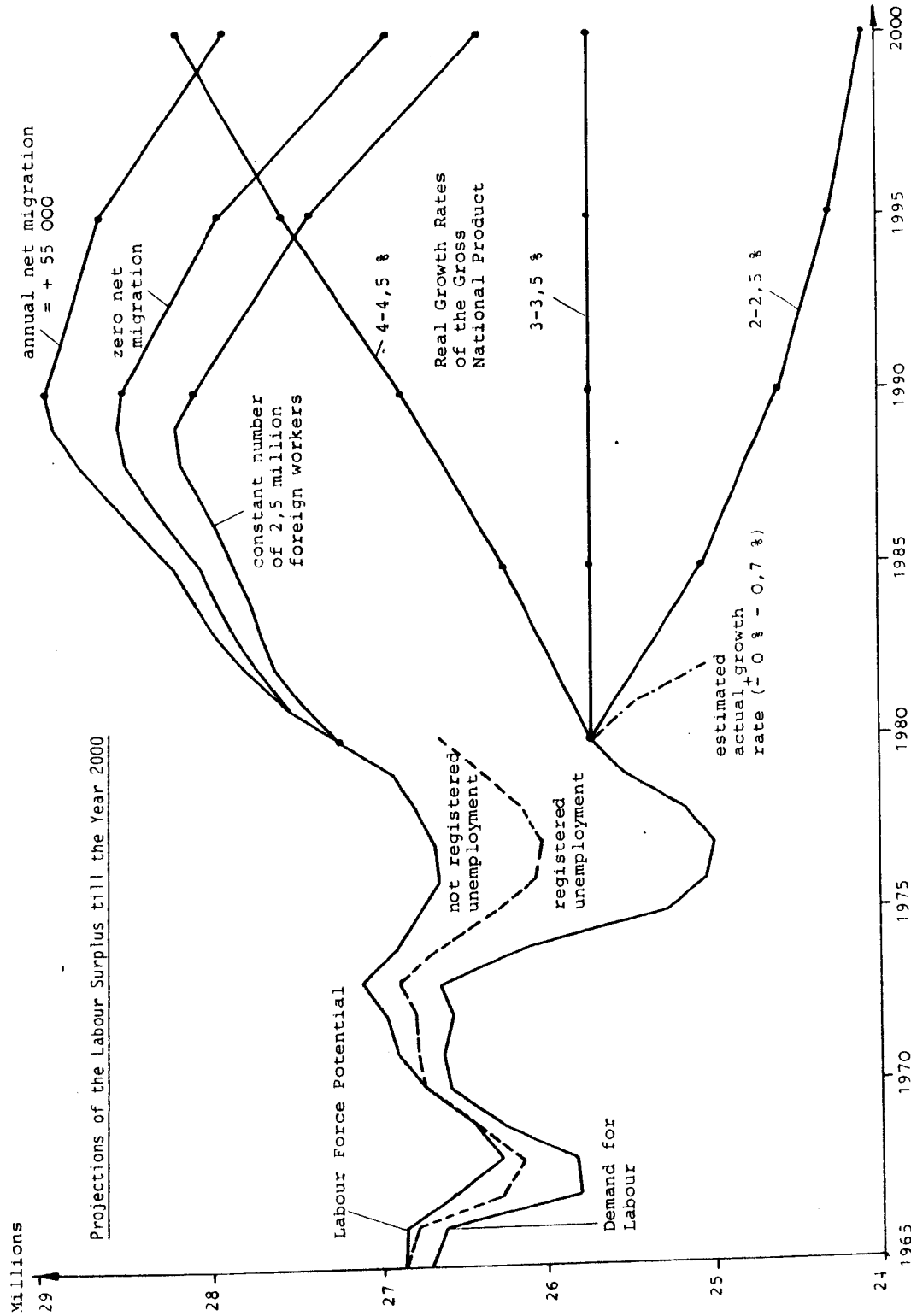
This approach of explaining the employment parameter can be justified by theoretical arguments as well as by empirical evidence. From a theoretical point of view it seems to be plausible that the higher the quantitative efficiency of the matching-process, as expressed by the magnitudes of α and β , the higher the congruency between the characteristics of each specific job demanded by an individual and the characteristics of each specific employee demanded by an investor. An alternative expression for "high congruency" is qualitative efficiency of the matching-process, and it seems to be plausible to assume a positive connection between the number of new jobs created by investment and the quality aspects of the efficiency of the matching process: The higher the congruency and quality of matching, the higher the probability that the workers remain employed for a long time interval in $[0, t]$ before changing their jobs or before becoming unemployed. Therefore the number of employed at the end of the period, $X(t)$, and the change of employment $\Delta X = X(t) - X(0)$ will be the higher the greater the proportion of durable matchings. Because α and β determine the number of matchings,

the proportion of durable matchings will be higher with increasing α and β . So the quality of matching is an increasing function of the quantity of matching.

An empirical argument for equation (53) is the following: In the Federal Republic of Germany approximately one out of four jobs becomes vacant and is re-occupied every year. This means that there were roughly speaking 60 million successful matchings in the period 1961-70, i.e. between the last two censuses. The unemployment rate was extremely low in this period, sometimes below 1%. Traditional economic interpretation of the low unemployment rates normally concentrates on the labour deficit between 1961 and 1970 which resulted in the immigration of 7,6 million people from other countries in this period. But a very important factor which is often overlooked is the following: German residence registration statistics show that the interregional mobility of employees was about 40% higher in the years of high employment in the sixties than it was in the years after 1970. The number of migrations between the different regions can therefore be interpreted as an indicator for the change in the number of successful matchings $M^*(t)$ since 60% of the interregional population movements were associated with occupational changes. From the enormous decline in the migration figures after 1970 it can be concluded that the increasing unemployment rates, which now are 10% and which will probably reach more than 15% in the second part of the eighties, are partly due to a decline in both the efficiency and the intensity of the matching-process. But here the question may rise as to whether the decline in the number of interregional movements is a result of the declining efficiency and intensity of the matching-process or a cause of the inefficiency in the matching-process. In the next section it will be shown that the decreasing migration figures are associated with a rise in the activity rate of married women after 1970. Since the probability of successful interregional matching for a couple which needs two new jobs when moving from one region to another is lower than the probability of successful matching for a single individual, inefficiency of matching is caused by the rising activity rates of married women, so that the inefficiency of the matching-process can be interpreted as a consequence of demographic change rather than being the cause of the rise in the activity rates and of the decline in the migration figures.

GRAPHS 2 and 3 illustrate the enormous change in employment and mobility before and after 1970. From GRAPH 3 it is evident that more than 50% of the job deficit or labour surplus is caused by demographic size factors.

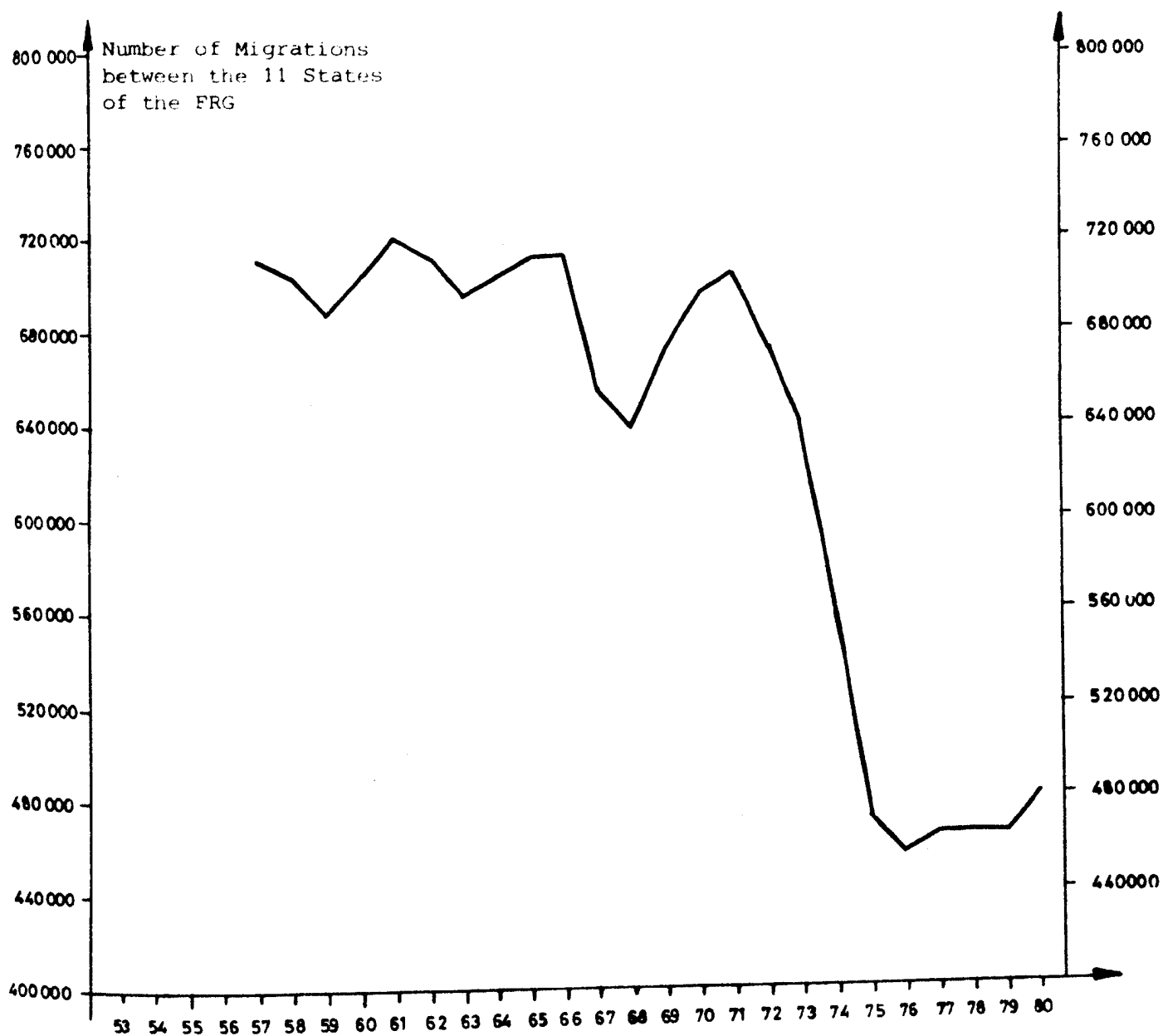
GRAPH 3



Source: W. Klauder, P. Schnur and M. Thon: "Wachstum und Arbeitsmarkt - Perspektiven 1980-2000".
In: Quintessenzen aus der Arbeitsmarkt- und Berufsforschung, 2. Nachtrag zu Quint A81,
Institut für Arbeitsmarkt- und Berufsforschung, Nürnberg, 1982.

GRAPH 2

Declining Interregional Mobility as an Indicator for the
Number of Successful Matchings



5. Structural Effects and Size Effects of Demographic Change on the Intensity and Efficiency of the Matching-Process - Empirical Evidence

5.1 Proxy Variables for the Estimation of the Number of Matchings - The Effect of the Business Cycle

In the Federal Republic of Germany the annual number of matchings is estimated to be about 10 million including

5 million intra-firm matchings and
5 million inter-firm matchings.

This is a rough estimation¹⁾. Exact figures for the annual number of matchings are available only for a special sub-group of all matchings, namely for those who obtained or changed their jobs with the assistance of the Federal Employment Agency. This agency has offices in 142 so called "Arbeitsamtsbezirke" which can be regarded as regional labour markets, and which cover the whole territory of the Federal Republic. The purpose of these offices is to fill the vacancies registered with them by the employers.

Figures for the officially registered number of matchings are given in TABLE 4. The annual figures vary between 3 million and 1,9 million, exhibiting a steady decline from 1970 corresponding to the growing number of unemployed. The officially registered number of the annual matchings is about 40% to 60% of the total number of the inter-firm matchings (5 million).

Another proxy variable for the number of matchings is the number of migrations between the 11 states of the Federal Republic. As most inter-state migrations involve a distance of 100 km or more, the proportion of

1) L. Reyher/D. Mertens: Zum Beschäftigungsproblem in den nächsten Jahren. In: Gewerkschaftliche Monatshefte, Vol. 11, 1977.

T A B L E 4

Development of the Number of Matchings Registered by
the Federal Institute of Labour 1964 - 1980

Year	Number of Matchings ¹⁾ in 1000			Rate of Change (total) %	Ratio female/total %
	total	male	female		
1964	2 949	2 175	774	--	26.2
1965	2 752	2 024	728	- 6.7	26.5
1966	2 653	1 906	747	- 3.6	28.2
1967	2 694	1 955	739	1.5	27.4
1968	2 911	2 149	761	8.1	26.1
1969	2 982	2 209	772	2.4	25.9
1970	2 988	2 213	775	0.2	25.9
1971	2 706	1 961	745	- 9.4	27.5
1972	2 583	1 855	728	- 9.5	28.2
1973	2 653	1 924	728	2.7	27.4
1974	2 239	1 614	624	-15.6	27.9
1975	2 127	1 529	598	- 5.0	28.1
1976	2 327	1 689	639	9.4	27.5
1977	2 290	1 664	626	- 1.6	27.3
1978	2 138	1 546	592	- 6.6	27.7
1979	2 111	1 530	581	- 1.3	27.5
1980	1 905	1 367	538	- 9.8	28.2

1) Number of placings registered officially by the "Bundesanstalt für Arbeit" (Federal Institute of Labour, Nürnberg)
Source: "Amtliche Nachrichten der Bundesanstalt für Arbeit - Arbeitsstatistik 1980 - Jahreszahlen", Vol. 29, Nürnberg 1981.
Ed.: Federal Institute of Labour, Nürnberg

residence changes which are combined with a change of job can be estimated to be at least 54%¹⁾.

The fluctuations in the number of matchings around the trend are obviously due to the cyclical variations of the GNP. In this analysis the cyclic variations are not examined. The development of the declining trend itself is examined in order to reveal demographic factors which can be expected to cause the steady downward shift of the trend.

In the theoretical analysis of the previous sections two broad classes of matchings were distinguished, renewal matchings and permanent stock matchings. In the following sections demographic factors are examined which effect both classes of matchings. The analysis concentrates on two special kinds of demographic factors whose effects are denoted as size effects and structural effects.

1) H. Birg: Zum Stand der Wanderungsforschung in der Bundesrepublik Deutschland. In: S. Rupp (ed.): Demographische Forschung heute - Bestandsaufnahme und Perspektiven. Jahrestagung der Deutschen Gesellschaft für Bevölkerungswissenschaft, Wiesbaden 1981, p. 129.

5.2 Size Effects

In this section the effects of the size of a labour market on the efficiency of the matching-process will be discussed on a theoretical basis. The empirical relevance of the arguments may not be important at the present time because many countries, e.g. the Federal Republic of Germany exhibit growing national labour markets till the end of the eighties associated with a decline in population. But at the end of the century the picture will be very different: All of the 160 regional labour markets of the Federal Republic will then exhibit a rapidly declining size due to the permanent birth deficit since 1971¹⁾.

Consider a regional labour market with n members of the labour force and n jobs. For simplicity it is assumed that all members of the labour force are employed and all jobs are occupied. Although there are no unemployed and no vacant jobs, matching can be very intensive: As pointed out in the previous section, in the Federal Republic, for example, one out of four employees changes his job every year.

Let it be assumed that the employees have the same profession but different abilities and characteristics which make them different with respect to their suitability for the jobs available. The individuals are numbered in such a way that individual 1 has the highest qualification, individual 2 the second best and so on. Let it be assumed that the jobs 1, ..., n are also different and that the employees

1) Figures for the labour surplus and for the labour deficit of the regional labour markets are published in: H. Birg, "Die Entwicklung der Bevölkerung und des Erwerbspotentials in den Regionen der Bundesrepublik Deutschland von 1975 bis 2000 - demographische Vorausberechnungen ohne Wanderungen. In: Gleichwertige Lebensbedingungen durch eine Raumordnungspolitik des mittleren Weges. Band 140 der Forschungs- und Sitzungsberichte der Akademie für Raumforschung und Landesplanung, Hannover. 1983.

rank the jobs according to their attractiveness, which may be a function of the income associated with each job as well as a function of various other factors.

Let it be assumed that every job becomes vacant within a certain period $[0, t]$. The individual 1 can choose among n alternative jobs within the period $[0, t]$ because individual 1 has the best qualifications. Assume that individual 1 chooses job 1, which is the most attractive. Then individual 2 can choose among $n-1$ alternatives, individual 3 among $n-2$ and so on:

employee	number of alternatives in the choice set of jobs
1	n
2	$n-1$
\vdots	\vdots
n	1

The total number Q of alternatives in all choice sets of all employees in the period $[0, t]$ is equal to the sum $Q = 1 + 2 + \dots + n$

$$(54) \quad Q = \frac{n(n+1)}{2}$$

and the average number of alternatives per employee is then

$$(55) \quad \frac{Q}{n} = (n + 1)/2$$

If the assumption is made that the probability for a single worker to find a suitable job is, on average, proportional to the number Q of alternatives in all choice sets, then the number of job vacancies V and of successful matchings M can both be assumed to be functions of Q ¹⁾:

1) $V = \beta Q$ is a better specification than the simple form $V = \beta n$, see equation (41).

$$\begin{aligned}
 V &= \beta Q \\
 (56) \quad M &= \alpha V \\
 &= \alpha \beta Q \\
 &= \alpha \beta \frac{n(n+1)}{2} \\
 0 &\leq \alpha, \beta \leq 1
 \end{aligned}$$

According to these assumptions the number of matchings M is determined by the size of the labour market, expressed by the number of members of the labour force n .

An important aspect of this result is that the matching function (56) is not linear and as a consequence the number of matchings per worker

$$(57) \quad \frac{M}{n} = \alpha \beta ((n+1)/2)$$

is higher the bigger the labour market.

Because an individual can choose not only among alternatives in his present region of residence, but also among alternatives on other labour markets, all members of the labour force in a region r can be regarded as potential emigrants and so the outcome (57) has considerable consequences especially for the theory of migration.

On the basis of this consideration it is possible to specify an out-migration function for regions or cities which can be empirically tested. The test of the out-migration function can be interpreted as an indirect testing procedure for the matching function (56), which cannot be tested directly because data for measuring the variable M are normally not available.

Let it be assumed that all members n^r of the labour force in a region r are potential emigrants, denoted by the symbol ${}^*E^r$. Then the effective number of out-migrations from region r to other regions in the country is the difference between ${}^*E^r$ and the number of members of the labour force who remain on the labour market in r . If it is assumed that the number of those remaining is proportional to the number of matchings

M on the labour market in r an out-migration function can be specified as:

$$(58) \quad E^r = E^r(*E^r - M^r)$$

where E^r is the number of individuals who migrate from region r or city r to other cities or regions within the country. Assuming a linear form

$$(59) \quad E^r = a_0 + a_1 n^r - a_2 M^r$$

and by substituting (56) into (59):

$$(60) \quad E^r = a_0 + a_1 n^r - a_2 \alpha \beta \frac{(n^r)^2}{2} + n^r$$

Equation (60) can be tested by the use of population figures if it is assumed that the activity rates n^r/p^r for regions or for cities respectively are of similar size (p^r denotes the number of inhabitants):

$$(61) \quad E^r = b_0 + b_1 p^r - b_2 \alpha \beta \frac{(p^r)^2}{2} + p^r$$

$$b_0, b_1, b_2 = \text{constant}$$

or:

$$(62) \quad E^r = c_0 + c_1 p^r - c_2 (p^r)^2$$

with $c_0 (=b_0), c_1, c_2 = \text{constant}$

$$c_1 = b_1 - \frac{b_2 \alpha \beta}{2} \text{ and } c_2 = b_2 \alpha \beta / 2$$

An OLS-estimation of equation (62) on the basis of the migration figures and population figures for all 58 cities of the FRG with more than 100,000 inhabitants in 1970 gave the following result ($u^r =$ error term, t - values in brackets):

$$(63) \quad E^r = -380.5 + 604.4 P^r - 0.959 (P^r)^2 + u^r ; \quad r = 1, \dots, 58$$

$$(11.6) \quad (-3.55)$$

$$\rho^2 = 0.923, \text{ corrected } \rho^2 = 0.920$$

$$F(2,55) = 330$$

P^r = population in units of 10,000 inhabitants.

As all t-values are above 2 at the 5 percent level and as the coefficient of determination is most significant, the hypothesis (63) can be accepted. Since (63) was deduced on the basis of the assumptions for the matching function (56) the test result of (63) can be interpreted as an indirect test of the correct specification of the matching function (56).

From (63) the number of out-migrations per person is

$$(64) \quad \frac{E^r}{P^r} = 604.4 - \frac{380.5}{P^r} - 0.959 P^r$$

Function (63) has a positive slope whereas function (64) has a negative slope over the range $P^r \geq 100,000$:

$$(63.1) \quad \frac{dE^r}{dP^r} = 604.4 - 1.92 P^r > 0; \quad P^r \text{ in units of 10,000}$$

$$(65) \quad \frac{d\left(\frac{E^r}{P^r}\right)}{dP^r} = -0.959 + \frac{380.5}{(P^r)^2} < 0$$

It can consequently be stated that (a) the number of out-migrations increases with increasing population and (b) the number of out-migrations per person decreases with increasing population. This outcome can be interpreted as a direct effect of the average level of matchings per member of the labour force, which is higher the greater the number of inhabitants:

$$(66) \quad \frac{M^r}{p^r} = \alpha\beta \frac{p^r + 1}{2}$$

A graphical representation of the theoretical matching function (56) and of the empirical out-migration function (63) is given in GRAPH 4. The results of the analysis of the size effects of demographic change can be summed up as follows:

- (1) The size of a labour market, measured by demographic variables as the number of the members of the labour force or the number of inhabitants, has important effects on the number of matchings: the bigger the labour market, the greater the number of matchings per member of the labour force. As a consequence the number of out-migrations is higher and the number of the out-migrations per person is lower the higher the number of inhabitants (or members of the labour force).
- (2) Diminishing regional labour markets will lead to a reduction in the number of successful matchings and to diminishing interregional migration flows. As a consequence also the employment rates of the regional labour markets will be influenced negatively by diminishing market sizes.

5.3 Structural Effects of Demographic Change

In this section the interrelations of the following processes are analyzed:

- (1) The diminishing growth rates of the gross national product which are associated with increasing unemployment.
- (2) The correlation between the increasing number of unemployed and the decreasing number of interregional migrations.
- (3) The parallelity of the number of interregional migrations and the number of successful matchings.
- (4) The increasing activity rates of married women which are interpreted to be one of the causes for the reduction of the number of interregional migrations and interregional matchings.

The number of migrations between the 11 states of the Federal Republic of Germany show a 30% reduction between 1971 and 1979 (TABLE 5 and GRAPH 2).

GRAPH 4

Theoretical Matching Function M^r and Empirical
Emigration Function E^r for Cities (r) with more
than 100 000 Inhabitants in the FRG

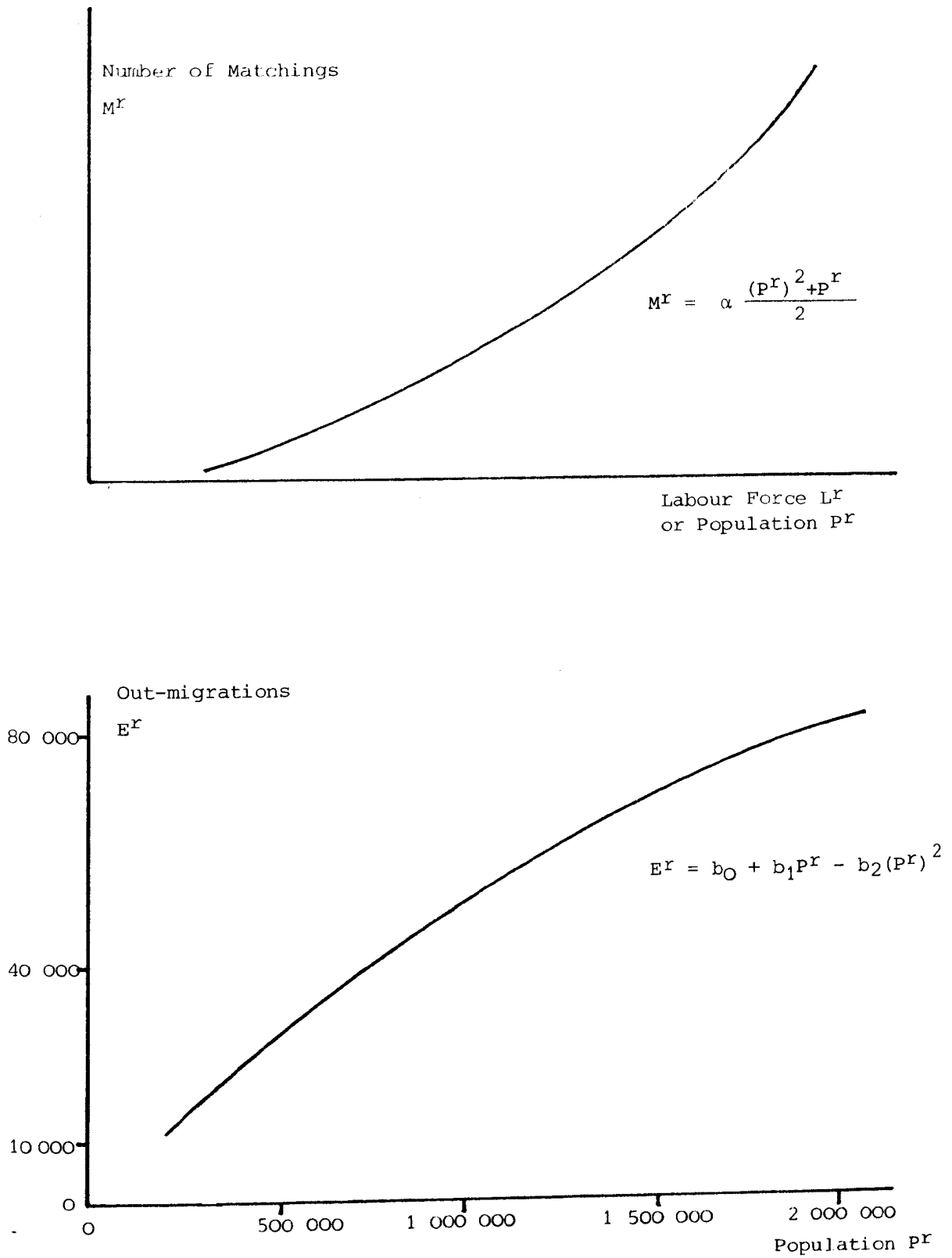


TABLE 5

Development of Migration and Unemployment in the FRG 1964-79

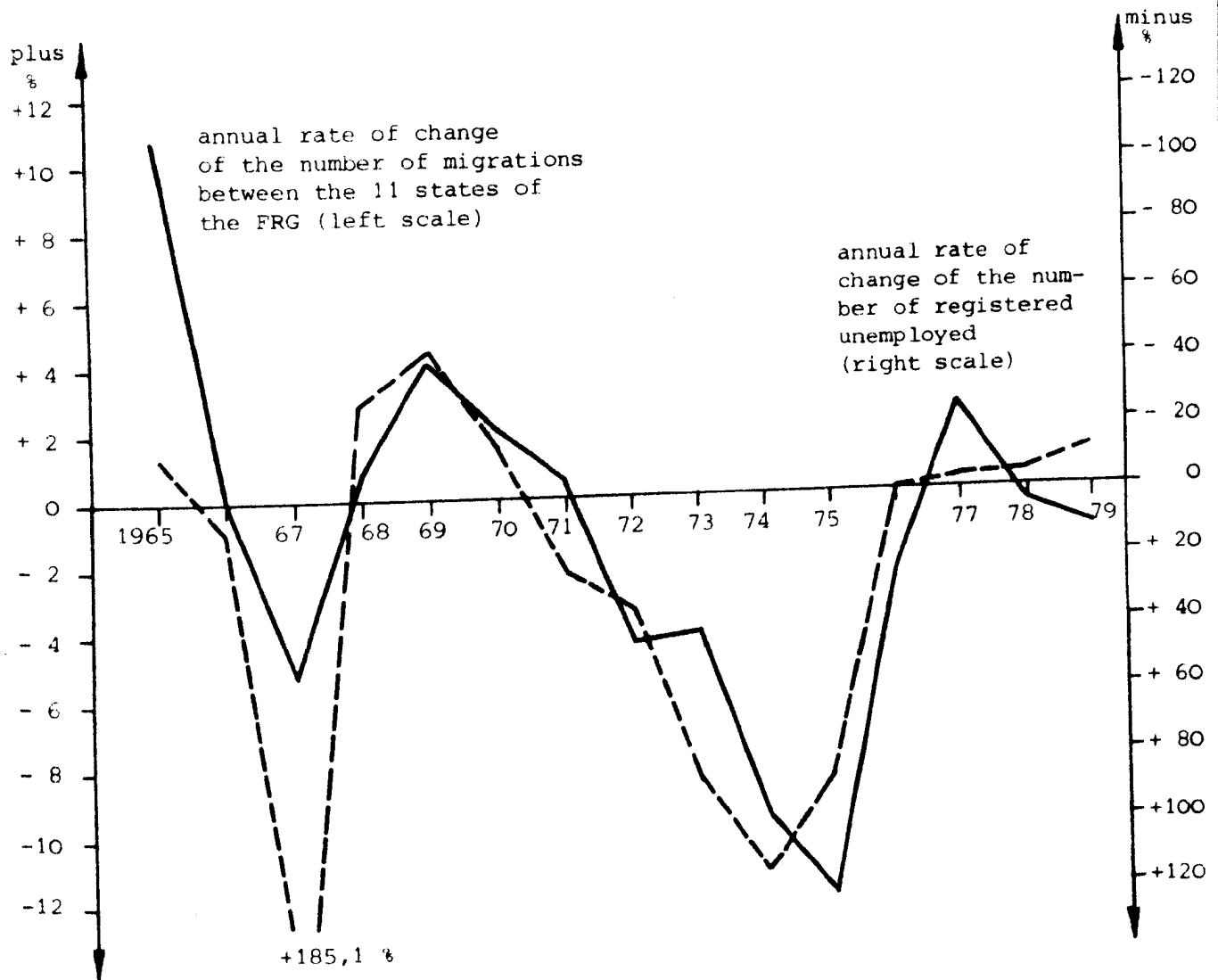
year	migrations ¹⁾ (in thousands)	number of unemployed ²⁾ (in thousands)	rate of change migration (in %)	rate of change unemployed (in %)
1964	992	169		
1965	1099	147	10.8	-13.0
1966	1100	161	0.1	9.5
1967	1042	459	- 5.3	185.1
1968	1050	323	0.8	-29.6
1969	1093	179	4.1	-44.6
1970	1118	149	2.3	-16.8
1971	1125	185	0.6	24.2
1972	1075	246	- 4.4	33.0
1973	1031	273	- 4.1	87.0
1974	930	582	- 9.8	113.2
1975	816	1074	-12.3	84.5
1976	796	1060	- 2.5	- 1.3
1977	817	1030	2.6	- 2.8
1978	814	993	- 0.4	- 3.6
1979	804	876	- 1.2	-11.8
1980		889		
1981		1272		
1982		1830		
1983		(2500)		

1) Number of migrations between the 11 states of the Federal Republic of Germany

2) Number of registered unemployed

GRAPH 5

The Reciprocal Parallelity of Migration and Unemployment
in the Federal Republic of Germany 1964-79*)



*Right scale = reversed left scale

What is the cause of the decline in interregional mobility? If the number of migrations would depend on the age distribution of the population, it could be expected that the changes of the age distribution are responsible for the decline in mobility. But as cohort analysis of mobility shows, the effect of the age structure is not responsible for this phenomenon.

The cohort analysis of mobility computes the number of migrations of persons of age a in year t in relation to the total number of persons of age a , both for a cohort born in year k :

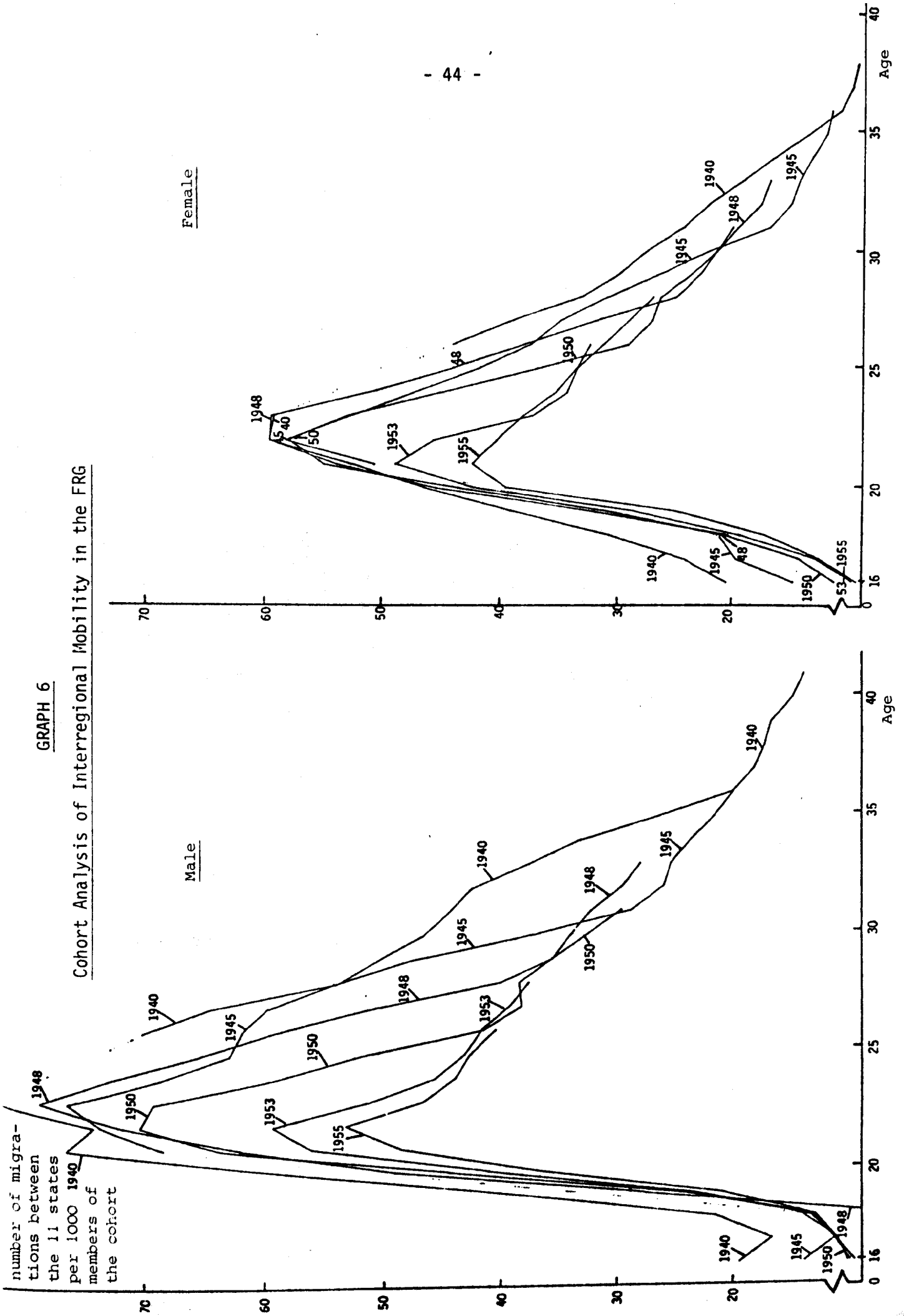
$$q_a^k(t) = \frac{E_a^k(t)}{P_a^k(t)} 1000 \quad ; \quad \text{for men and women respectively}$$

GRAPH 6 shows the development of the age and cohort specific mobility rates $q_a^k(t)$ for men and women. It can obviously be seen that all cohorts exhibit a considerable decline of mobility, and the younger the cohorts the greater the decline.

The decline of mobility is due to the diminishing growth rates of the GNP (GRAPH 7) but there is an additional factor which causes a reduction of mobility, namely the increase in the activity rates of married women (see TABLE 6). The link between mobility and activity rates is suspected to be an outcome of the factors which determine the probability of a job change. If the probability of a job change which is combined with a change of residence for an unmarried person i is p_i , then the probability of a job-change q_{ij} for married people i and j who want to change their residential location is less than p_i and less than p_j , namely $q_{ij} \approx p_i p_j$. About 60% of the number of migrations between the 11 states of the Federal Republic are combined with a job change. A rise in the activity rates of married women means that the number of potential migrants with probability p_i decreases and the number of potential migrants with probability $q_{ij} < p_i$ increases so that the overall probability for migration must diminish (GRAPH 2 above).

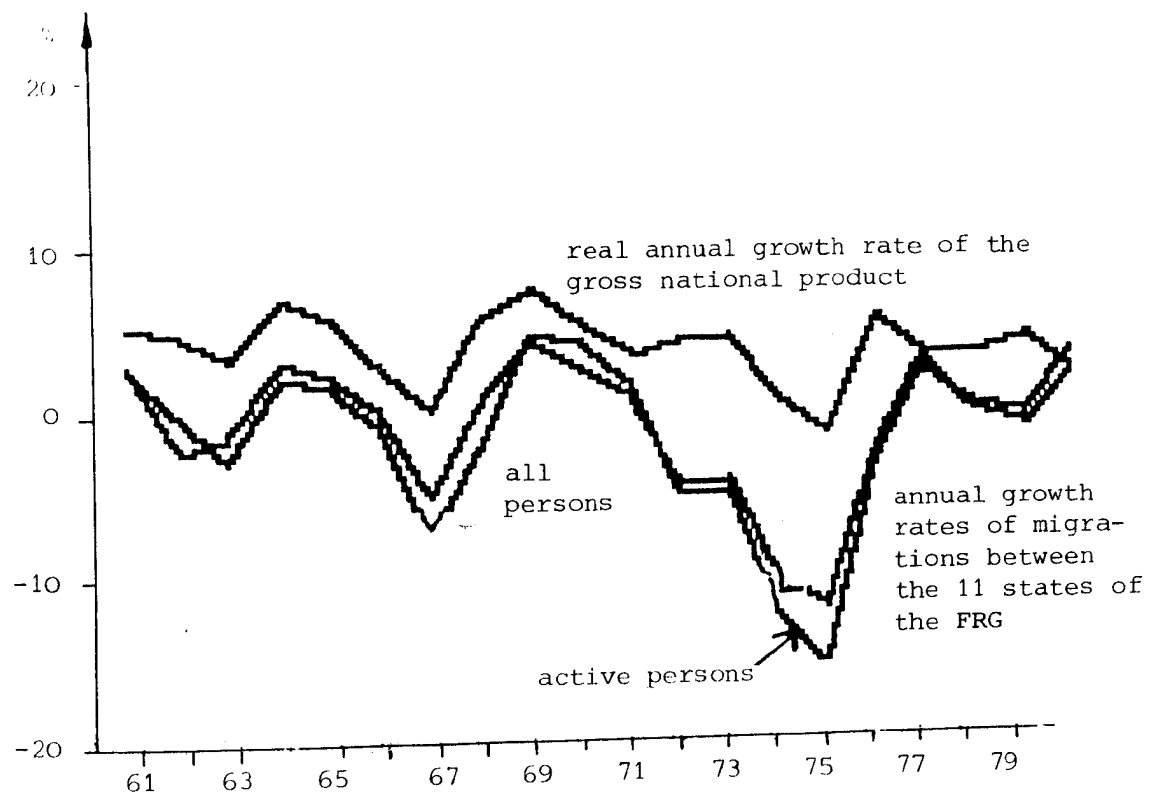
GRAPH 6

Cohort Analysis of Interregional Mobility in the FRG



GRAPH 7

Parallelity between Migration and Production



T A B L E 6

Age-specific Rates of Labor Force Participation of Married
Women in the Federal Republic of Germany (per 100)

year	15-20	20-25	25-30	30-35	35-40	40-45
1960	62.8	51.3	39.6	36.6	36.9	36.4
1961	62.8	52.5	40.4	36.0	37.2	37.7
1962	59.6	51.5	40.3	37.1	38.7	38.8
1963	56.9	52.2	40.4	36.8	37.2	38.4
1964	53.8	51.8	40.4	35.8	37.8	39.9
1965	53.7	51.7	40.8	36.5	38.3	40.6
1966	56.8	51.5	40.1	36.5	37.4	41.3
1967	54.8	50.4	39.8	35.2	37.2	40.0
1968	54.8	51.8	40.8	36.8	38.1	39.8
1969	58.7	52.4	42.3	37.8	39.0	40.5
1970	58.1	55.1	43.5	39.8	41.1	42.3
1971	56.7	56.0	46.5	40.6	41.4	42.8
1972	57.3	58.7	48.3	43.2	44.2	44.1
1973	55.5	61.0	50.3	45.0	45.3	45.5
1974	52.4	61.1	51.9	46.1	45.7	46.6
1975	57.9	62.1	51.2	47.1	45.9	46.5
1976	59.9	62.6	52.1	47.3	46.9	46.8
1977	55.8	63.4	52.9	48.4	48.0	48.8
1978	54.5	62.6	53.1	48.8	48.9	48.8
1979	56.1	61.1	53.8	49.8	49.4	50.0

Source: K.F. Zimmermann: The Dynamics of Women's Labour Force Participation and Fertility: A Simultaneous ARX-Model. Paper presented at the International Conference on the "Economic Consequences of Population Change in Industrialized Countries", Paderborn 1983.

As about 60% of the inter-state migrations are combined with a job change, mobility figures can be used as a proxy variable for the number of matchings:

$$M(t) \sim \sum_r \sum_s E^{rs}(t)$$

$$r \neq s$$

$$r, s = 1, \dots, 11 \text{ states}$$

The interpretation of mobility as a matching-indicator is supported by the strong correlation between the number of unemployed and the number of migrations. The graphical representation of the unemployment and migration figures in GRAPH 5 shows a strong reciprocal relation between the annual rates of change of employment and mobility: The higher the unemployment, the lower the mobility.

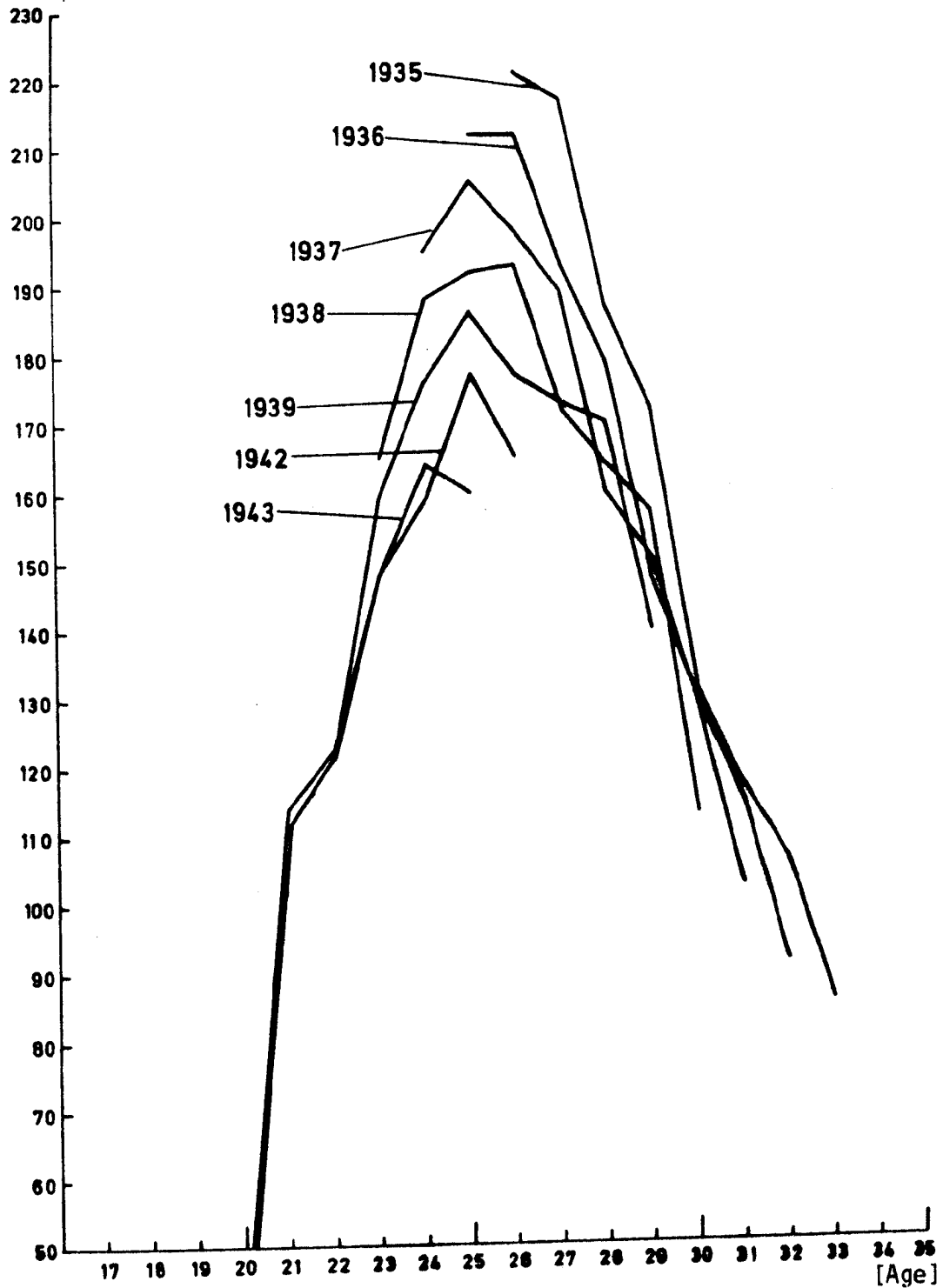
After 1976 relationship between female activity rates, mobility and unemployment is weakened by the acceleration of the decrease of the propensity of young people for marriage: Cohort analysis of the rates "number of persons of age a which marry for the first time in year t " in percent of the "number of unmarried persons of age a at the beginning of year t " shows that the cohort curves for younger cohorts are always below the curves for the elder cohorts - a picture which is similar to the cohort curves of mobility (GRAPH 6 and 8).

It can be concluded that at least in the period 1964-77 demographic variables such as the number of migrations and the activity rates of married women contributed to the decline in the number of matchings and to the increase in unemployment. The relationship between demographic and economic variables in the period 1964-77

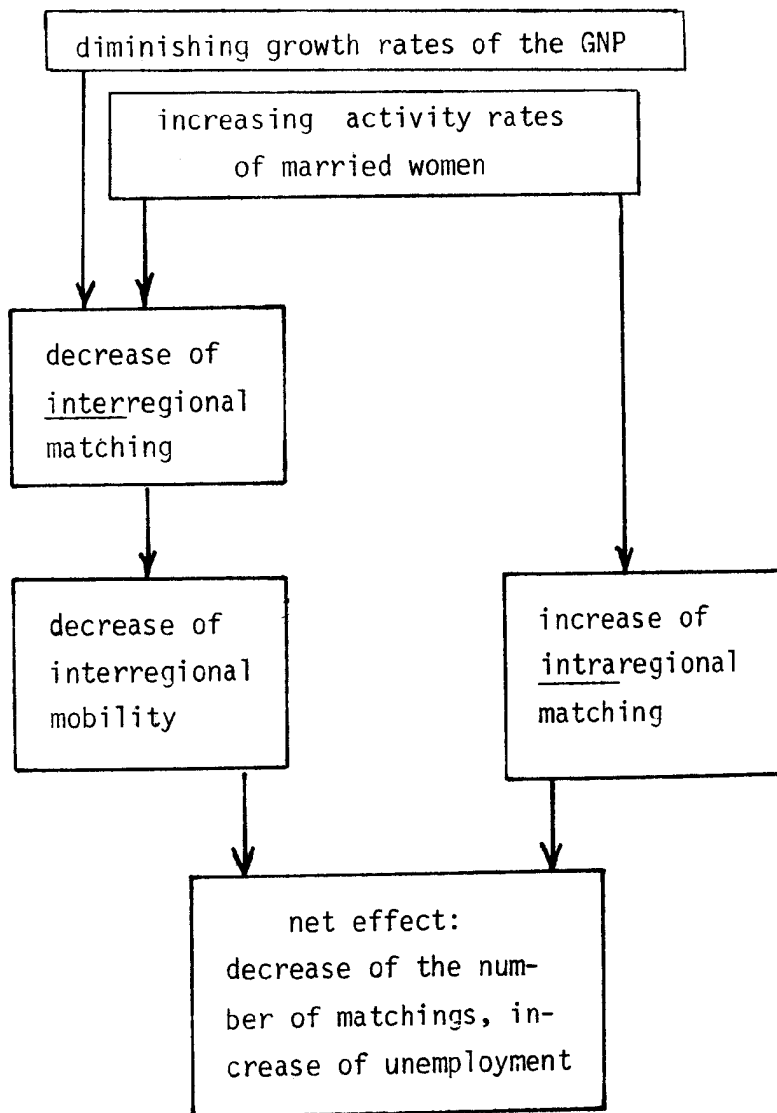
G R A P H 8

Cohort Analysis of Nuptiality

Number of First Marriages of
Men per 1000 Unmarried Men



may be virtualized by the following graph:



The influence of the growth rate of the GNP on the number of matchings is, in the first place, a cyclical one, whereas the influence of the rising activity rate of married women is not cyclical but based on a stable trend. The cyclical influence of the business cycle is stronger than the trend influence of the activity rate.

The increase of the activity rates of married women has two different effects on the number of matchings. The first effect causes a decrease of the number of the interregional matchings, the second causes an increase of the number of the intraregional matchings. The net effect is suspected to be negative.

6. Summary

The study starts with a macro-analysis of the effects of the growing size of the labour force on the balance between labour and jobs. The next step is a micro-analysis of the matching-process whose function is to attach individual members of the labour force to individual jobs. The interdependencies between the different components of change in the labour force and the different components of change in the stock of jobs are shown to be sources of the different types of the matching-process. Two classes of matching-processes are distinguished: renewal matching and permanent-stock-matching.

Demographic change influences renewal-matching as well as permanent-stock-matching. Size effects and structural effects of demographic change are distinguished and studied separately. In many industrialized countries demographic change is likely to be a major cause for the decrease in both the intensity and the efficiency of the matching-process. Growing unemployment figures and decreasing inter-regional matching and interregional mobility are interpreted to be principally outcomes of changes in demographic variables.